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Effect Algebras Which Can Be Covered by MV-Algebras

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We exhibit effect algebras which can be covered by MV-subalgebras. We show that any effect algebra E which satisfies the Riesz interpolation property (RIP) and the so-called difference-meet property (DMP) can be covered by blocks, maximal subsets of mutually strongly compatible elements of E, which are always MV-subalegbras. This result generalizes that of Riečanová who proved the same result for lattice-ordered effect algebras. We show that for effect algebras with only (RIP) the result in question can fail.

1. INTRODUCTION

Nowadays there exists a whole hierarchy of quantum structures (Dvurečenskij and Pulmannová, 2000): quantum logics, orthomodular lattices, orthomodular posets, orthoalgebras which correspond to two-valued, yes-no, events. In 1994 effect algebras entered quantum structures by Foulis and Bennett (1994) and they combine both algebraic and fuzzy set ideas of quantum measurement. They correspond to many-valued reasoning of quantum experiments and the most important example is $\mathcal{E}(H)$, the system of all effect operators, i.e., of all Hermitian operators A of a Hilbert space H such that $O \leq A \leq I$.

Effect algebras are equivalent to weak orthoalgebras of Giuntini and Greuling (1989) from 1989 and D-posets introduced by Kôpka and Chovanec (1994) in 1992. We recall that a partial algebra $E = (E; +, 0, 1)$ is said to be an *effect algebra* if, for all $a, b, c \in E$,

(EAi) $a + b$ is defined in E iff $b + a$ is defined, and in such the case $a + b = b + a$;

(EAii) $a + b$, $(a + b) + c$ are defined iff $b + c$ and $a + (b + c)$ are defined, and in such the case $(a + b) + c = a + (b + c)$;

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(EAiii) for any $a \in E$, there exists a unique element $a' \in E$ such that $a + a' = 1$;
(EAiv) if $a + 1$ is defined in E, then $a = 0$.

If we define $a \leq b$ iff there exists an element $c \in E$ such that $a + c = b$, then $\leq$ is a partial ordering, and we write $c := b - a$.

For example, if (G, u) is an Abelian unital po-group with a strong unit u, and if $\Gamma(G, u) := \{g \in G : 0 \leq g \leq u\}$ is endowed with the restriction of the group addition $+$, then $(\Gamma(G, u); +, 0, u)$ is an effect algebra. An effect algebra E is an *orthoalgebra* if $a + a \in E$ entails $a = 0$.

MV-algebras entered mathematics by Chang (1958) in 1958.

We recall that an MV-*algebra* is an algebra $M := (M; \oplus, \odot, ^*, 0, 1)$ of type (2,2,1,0,0) such that, for all $a, b, c \in M$, we have

(MVi) $a \oplus b = b \oplus a$;
(MVii) $(a \oplus b) \oplus c = a \oplus (b \oplus c)$;
(MViii) $a \oplus 0 = a$;
(MViv) $a \oplus 1 = 1$;
(MVv) $(a^*)^* = a$;
(MVvi) $a \oplus a^* = 1$;
(MVvii) $0^* = 1$;
(MVviii) $(a^* \oplus b)^* \oplus b = (a \oplus b^*)^* \oplus a$.

If we define a partial operation $+$ on M in such a way that $a + b$ is defined in E iff $a \leq b^*$, then $a + b := a \oplus b$, then $(M; +, 0, 1)$ is an effect algebra.

MV-algebras have appeared in effect algebras in many natural ways: Mundici (1986) showed that starting from an AF C^*-algebras we can obtain countable MV-algebras, and conversely, any countable MV-algebra can be derived in such a way. Bennett and Foulis (1995) introduced Φ-symmetric effect algebras which are exactly MV-algebras, and also Boolean D-posets of Chovanec and Kôpka (1992) are MV-algebras.

MV-algebras play a similar role in effect algebras as Boolean algebras in orthomodular posets—they describe maximal sets of mutually (strongly) compatible elements. Moreover, Riečanová (2000a) recently proved an important result that each lattice ordered effect algebra can be covered by MV-subalgebras which form blocks.

In this paper, we extend this result for effect algebras with the Riesz interpolation property (RIP) and with the decomposition-meet property. Such effect algebras are not necessary lattice-ordered, but every lattice effect algebra satisfies our conditions.

We recall that Jenča studied blocks of mutually compatible elements satisfying the Riesz decomposition property. However, such blocks are not necessary MV-algebras.

Finally, we illustrate our approach by examples.

2. EFFECT ALGEBRAS WITH THE RIESZ INTERPOLATION PROPERTY

Let E be an effect algebra. We say that E satisfies (i) the *Riesz interpolation property*, (RIP) for short, if, for all x_1, x_2, y_1, y_2 in E, $x_i \leq y_j$ for all i, j implies there exists an element $z \in E$ such that $x_i \leq z \leq y_j$ for all i, j; (ii) the *Riesz decomposition property*, (RDP) for short, if $x \leq y_1 + y_2$ implies that there exist two elements $x_1, x_2 \in$ with $x_1 \leq y_1$ and $x_2 \leq y_2$ such that $x = x_1 + x_2$.

We recall that (1) if E is a lattice, then E has trivially (RIP); the converse is not true as we see below. (2) E has (RDP) iff, (Dvurečenskij and Pulmannová, 2000; Lem 1.7.5), $x_1 + x_2 = y_1 + y_2$ implies there exist four elements $c_{11}, c_{12}, c_{21}, c_{22} \in E$ such that $x_1 = c_{11} + c_{12}$, $x_2 = c_{21} + c_{22}$, $y_1 = c_{11} + c_{21}$, and $y_2 = c_{12} + c_{22}$. (3) (RDP) implies (RIP), but the converse is not true (e.g. if $E = L(H)$, then E is a complete lattice but without (RDP)).

We recall that a poset $(E; \leq)$ is an *antilattice* if only comparable elements of E have a supremum (infimum). It is clear that any linearly ordered poset is an antilattice.

There exists an effect algebra with (RIP) which is not a lattice:

Example 2.1. Let G be the additive group $\mathbb{R}^2$ with the positive cone of all (x, y) such that either $x = y = 0$ or $x > 0$ and $y > 0$. Then $u = (1, 1)$ is a strong unit for G. The effect algebra $E = \Gamma(G, u)$ is an antilattice having (RIP) and (RDP) but E is not a lattice.

Two elements a and b of an effect algebra E are said to be (i) *compatible* and write $a \leftrightarrow b$ if there exist three elements $a_1, b_1, c \in E$ such that $a = a_1 + c$, $b = b_1 + c$ and $a_1 + b_1 + c \in E$, and (ii) *strongly compatible* and we write $a \stackrel{c}{\longleftrightarrow} b$ if there are three elements $a_1, b_1, c \in E$ such that $a = a_1 + c$, $b = b_1 + c$, $a_1 \wedge b_1 = 0$ and $a_1 + b_1 + c \in E$.

We recall that (i) if $a \stackrel{c}{\longleftrightarrow} b$, then $a \leftrightarrow b$; (ii) $a \leftrightarrow b$ ($a \stackrel{c}{\longleftrightarrow} b$) implies $b \leftrightarrow a$ ($b \stackrel{c}{\longleftrightarrow} a$); (iii) $0 \stackrel{c}{\longleftrightarrow} a \stackrel{c}{\longleftrightarrow} 1$; (iv) if $a \leq b$, then $a \stackrel{c}{\longleftrightarrow} b$, ($b = (b - a) + a$, $a = 0 + a$).

We show that if $a \stackrel{c}{\longleftrightarrow} b$, then the corresponding elements a_1, b_1, c are uniquely determined in E with (RIP). If a and b are only compatible, there is possible to find more triples of a_1, b_1, c satisfying $a = a_1 + c$, $b = b_1 + c$ and $a_1 + b_1 + c \in E$.

Proposition 2.2. *Let an effect algebra E satisfy (RIP). If $a = a_1 + c$, $b = b_1 + c$ with $a_1 \wedge b_1 = 0$ and $a_1 + b_1 + c \in E$, then $a \wedge b = c$, $a \vee b = a_1 + b_1 + c$.*

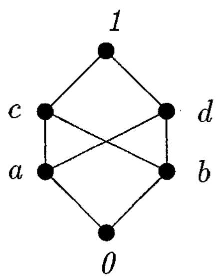
Proof: We have $c \leq a, b$. If $d \leq a, b$, there exists $d_0 \in E$ such that $c, d \leq d_0 \leq a, b$. Hence $d_0 - c \leq a - c = a_1$, $d_0 - c \leq b - c = b_1$, so that $d_0 - c \leq a_1 \wedge b_1 = 0$.

Put $u = a_1 + b_1 + c$. Then $u \geq a, b$. Assume $e \geq a, b$. There exists $e_0 \in E$ such that $u, c \geq e_0 \geq a, b$. Then $u - e_0 \leq u - a = b_1$ and $u - e_0 \leq u - b = a_1$ so that $u - e_0 = 0$ and $u = e_0 \leq e$. $\square$

The following example is from Riečanová (2000b).

Example 2.3. Let $E = \{0, a, b, c, d, 1\}$, where the addition $+$ is defined in the table.

$+$	0	a	b	c	d	1
0	0	a	b	c	d	1
a	a	d	c	1	$\times$	$\times$
b	b	c	d	$\times$	1	$\times$
c	c	1	$\times$	$\times$	$\times$	$\times$
d	d	$\times$	1	$\times$	$\times$	$\times$
1	1	$\times$	$\times$	$\times$	$\times$	$\times$
$'$	0	a	b	c	d	1
	1	c	d	a	b	0



Then E is an effect algebra which is not a lattice and without (RIP), but all elements of E are strongly compatible and e.g. $c \overset{c}{\longleftrightarrow} b$ and $c \vee d \in E$ but $c \wedge d \notin E$.

Proposition 2.4. (1) *If* $a \vee b \in E$, *then*

$$((a \vee b) - a) \wedge ((a \vee b) - b) = 0.$$

(2) *If* $a \wedge b \in E$, *then*

$$(a - (a \wedge b)) \wedge (b - (a \wedge b)) = 0.$$

Proof: (1) Let $x \leq (a \vee b) - a$ and $x \leq (a \vee b) - b$. Then $x + a \leq a \vee b$ and $x + b \leq a \vee b$, that is $a \leq (a \vee b) - x$ and $b \leq (a \vee b) - x$. Hence $a \vee b \leq (a \vee b) - x$ which yields $x = 0$.

In a similar way we prove (2). $\square$

Proposition 2.5. *Let* $a \leftrightarrow b$ *and* $a \wedge b \in E$. *Then* $a \overset{c}{\longleftrightarrow} b$.

Proof: Let $a = a_1 + c$ and $b = b_1 + c$ with $a_1 + b_1 + c \in E$. Then $c \leq a \wedge b$ and $a_1 = a - c \geq a - (a \wedge b)$ and $b_1 = b - c \geq b - (a \wedge b)$. Therefore, $a_1 = a - c = ((a - c) - (a - (a \wedge b)) + (a - (a \wedge b)) = (a \wedge b) - c) + (a - (a \wedge b))$. Similarly $b = b - c = ((a \wedge b) - c) + (b - (a \wedge b))$. Hence $a = a_1 + c = (a - (a \wedge b)) + ((a \wedge b) - c) + c$ and $b = b_1 + c = (b - (a \wedge b)) + ((a \wedge b) - c) + c$, and finally, $a_1 + b_1 + c = (a - (a \wedge b)) + a \wedge b + (b - (a \wedge b)) \in E$. Applying (2) of Proposition 2.4, we have $a \overset{c}{\longleftrightarrow} b$. □

Proposition 2.6. *If $a \leftrightarrow b (a \overset{c}{\longleftrightarrow} b)$, then $a' \leftrightarrow b' (a' \overset{c}{\longleftrightarrow} b')$.*

Proof: Set $u = a_1 + b_1 + c$. Then $1 = a_1 + b_1 + c + u'$ which yields $a_1' = b_1 + u'$ and $b' = a_1 + u'$. □

Proposition 2.7. *Let $a \leftrightarrow b$ and $a \vee b \in E$. Then $a \overset{c}{\longleftrightarrow} b$.*

Proof: We have $(a \vee b)' = a' \wedge b' \in E$. Proposition 2.6 and Proposition 2.5 gives $a' \overset{c}{\longleftrightarrow} b'$, consequently, $a \overset{c}{\longleftrightarrow} b$. □

As a consequence of Proposition 2.5 or Proposition 2.6 we have that in lattice effect algebras the compatibility and the strong compatibility coincide. We note that if E does not satisfy (RIP), then the existence of a join (or of a meet) of strongly compatible elements does not entail the existence of a meet (a join), see Example 2.3.

Theorem 2.8. *Let an effect algebra E satisfy* (*RIP*)*. The following statements are equivalent.*

(i) $a \leftrightarrow b$ *and* $a \wedge b \in E$.
(ii) $a \leftrightarrow b$ *and* $a \vee b \in E$.
(iii) $a \overset{c}{\longleftrightarrow} b$.
(iv) $a \vee b, a \wedge b \in E$ *and* $(a \vee b) - b = a - (a \wedge b)$.
(v) $a \vee b, a \wedge b \in E$ *and* $(a \vee b) - a = b - (a \wedge b)$.

Proof: The equivalence of (i)–(iii) follows from Propositions 2.5 and 2.7. Assume $a \overset{c}{\longleftrightarrow} b$. Then by Proposition 2.2, $a \vee b, a \wedge b \in E$. Hence $a \vee b = a_1 + b_1 + c$ and $(a \vee b) - b = a_1 = a - (a \wedge b)$.

Let (iv) hold. Then $a = (a - (a \wedge b)) + (a \wedge b)$ and $b = (b - (a \wedge b)) + (a \wedge b)$. Hence $a \vee b = (a - (a \wedge b)) + b = (a - (a \wedge b)) + (b - (a \wedge b)) + (a \wedge b) \in E$. Using (2) of Proposition 2.4, we see that $a \overset{c}{\longleftrightarrow} b$.

The equivalence of (iii) and (v) is similar. □

Proposition 2.9. *Let an effect algebra E satisfy (RIP). Assume $b \overset{c}{\longleftrightarrow} a_1$, $b \overset{c}{\longleftrightarrow} a_2$ and $a_1 \vee a_2 \in E$. Then $b \overset{c}{\longleftrightarrow} (a_1 \vee a_2)$ and*

$$b \wedge (a_1 \vee a_2) = (b \wedge a_1) \vee (a \wedge a_2).$$

Proof: Let $a = a_1 \vee a_2 \in E$. By Proposition 2.2, $b \wedge a_1, b \wedge a_2 \in E$. We have $b \wedge a_1, b \wedge a_2 \le a, b$. Choose any element $b_0 \in E$ such that $b \wedge a_1, b \wedge a_2 \le b_0 \le a, b$; due to (RIP) such an element exists.

Claim 1. $b \leftrightarrow a$.

It is clear that $a = (a - b_0) + b_0$ and $b = (b - b_0) + b_0$. On the other hand, we have $a_i \le (b - (b \wedge a_i))' \le (b - b_0)'$ so that $a \le (b - b_0)'$ which gives $E \ni a + (b - b_0) = (a - a_0) + (b - b_0) + b_0 \in E$.

Claim 2. $(b - (b \wedge a_1)) \wedge (b - (b \wedge a_2)) = b - b_0$.

It is evident that $b - (b \wedge a_i) \ge b - b_0$ for $i = 1, 2$. Let $d \le b - (b \wedge a_i)$ for $i = 1, 2$. By Theorem 2.8. $d \le b - (b \wedge a_i) = (b \vee a_i) - a_i$. Then by Claim 1, $d + a_i \le b \vee a_i \le (b - b_0) + a$, so that $a_i \le ((b - b_0) + a) - d$ and $a \le ((b - b_0) + a) - d$ which gives $d + a \le (b - b_0) + a$ and $d \le b - b_0$.

Claim 3. $(b - b_0) \wedge (a - b_0) = 0$.

Assume $z \le b - b_0$ and $z \le a - a_0$. Then $z + b_0 \le b$ and $z + b_0 \le a$. Moreover, $b \wedge a_i \le z + b_0 \le b, a$ for $i = 1, 2$. Using Claims 1 and 2 for the element $z + b_0$, we have $b - b_0 = b - (z + b_0)$, i.e., $z = 0$.

Claim 4. $a \overset{c}{\longleftrightarrow} b$ and $a \vee b, a \wedge b \in E$.

It follows from Claim 3 and Proposition 2.2.

Claim 5. $b \wedge (a_1 \vee a_2) = (b \wedge a_1) \vee (b \wedge a_2)$.

It is clear that $b \wedge a \ge b \wedge a_1, b \wedge a_2$. Assume $b \wedge a_1, b \wedge a_2 \le y$. Then $b \wedge a_1, b \wedge a_2 \le y, b_0$ so that there exists an element $y_0 \in E$ such that $b \wedge a_i \le y_0 \le y, b_0$ for $i = 1, 2$. Then $b - y_0 \le b - (b \wedge a_i)$. By Claim 2, we have $b - y_0 \le \bigwedge_{i=1}^{2}(b - (b \wedge a_i)) = b - b_0$, so that $b_0 \le y_0 \le y$ which finishes the proof. □

Proposition 2.10. *Let an effect algebra E satisfy (RIP). Assume $b \overset{c}{\longleftrightarrow} a_1$, $b \overset{c}{\longleftrightarrow} a_2$ and $a_1 \wedge a_2 \in E$. Then $b \overset{c}{\longleftrightarrow} (a_1 \wedge a_2)$, and*

$$b \vee (a_1 \wedge a_2) = (b \vee a_1) \wedge (b \vee a_2).$$

Proof: Because of Proposition 2.6, we have $b' \overset{c}{\longleftrightarrow} a_i'$ for $i = 1, 2$, and $(a_1 \wedge a_2)' = a_1' \vee a_2' \in E$. Applying Proposition 2.9, we have $b' \overset{c}{\longleftrightarrow} (a_1' \vee a_2')$ so that $b \overset{c}{\longleftrightarrow} (a_1 \wedge a_2)$. Hence $b' \wedge (a_1' \vee a_2') = (b' \wedge a_1') \vee (b' \wedge a_2')$ which gives $b \vee (a_1 \wedge a_2) = (b \vee a_1) \wedge (b \vee a_2)$. □

Proposition 2.11. *In an antilattice effect algebra E with (RIP) $a \overset{c}{\longleftrightarrow} b$ if and only if $a \wedge b \in E$.*

Proof: One direction follows from Proposition 2.2. Assume now $a \wedge b \in E$. Then either $a \le b$ or $b \le a$, which gives in the first case $a = 0 + a$, $b = (b - a) + a$, and similarly for the second case. □

3. EFFECT ALGEBRAS WITH THE DIFFERENCE-MEET PROPERTY

We say that an effect algebra E satisfies the *difference-meet property*, (DMP) for short, if $x \le y$, $x \wedge z \in E$ and $y \wedge z \in E$ imply $(y - x) \wedge z \in E$. For example every lattice-ordered effect algebra satisfies the difference-meet property. On the other hand, Example 2.1 gives an example of an antilattice effect algebra with (RIP) and (RDP) where the differnce-meet property fails. In addition, it can be shown that there exist two elements $a, b \in E$ such that $a \overset{c}{\longleftrightarrow} b$, $a \leftrightarrow b'$ but $a \overset{c}{\not\longleftrightarrow} b'$, and $b \overset{c}{\not\longleftrightarrow} b'$.

Proposition 3.1. *Let an effect algebra E satisfy (RIP) and (DMP).*

(i) *If $a \overset{c}{\longleftrightarrow} b$, then $a \overset{c}{\longleftrightarrow} b'$.*
(ii) *If $a \overset{c}{\longleftrightarrow} b$, $a \overset{c}{\longleftrightarrow} c$ and $b \le c$, then $a \overset{c}{\longleftrightarrow} (c - b)$.*

Proof: (i) Since $a \overset{c}{\longleftrightarrow} 1$ and $b \le 1$, we have $a \wedge b \in E$, $a \wedge 1 \in E$, (DMP) entails $a \wedge (1 - b) = a \wedge b' \in E$. On the other hand, we have $a = a_1 + c$, $b = b_1 + c$, where $u := a_1 + b_1 + c \in E$ and $a_1 \wedge b_1 = 0$. Then $1 = a_1 + b_1 + c + u'$, so that $b' = a_1 + u'$, i.e., $a \leftrightarrow b'$. Applying Proposition 2.5, we have $a \overset{c}{\longleftrightarrow} b'$.

(ii) First we show that $a \leftrightarrow (c - b)$. By Proposition 2.2, $a \wedge b, a \wedge c, a \vee b, a \vee c \in E$. Then $a \wedge b \le a \wedge c$, and there exists an element $w \in E$ such that $(a \wedge b) + w = a \wedge c$. Therefore, by Proposition 2.2, $a \vee b = (b - (a \wedge b)) + (a - (a \wedge b)) + a \wedge b \le a \vee c = (a - (a \wedge c)) + (c - (a \wedge c)) + a \wedge c$. Since $a = (a \wedge b) + (a - (a \wedge b)) = (a \wedge c) + (a - (a \wedge c))$, we have $b - (a \wedge b) \le c - (a \wedge c)$. There exists another element $v \in E$ such that $(b - (a \wedge b)) + v = c - (a \wedge c)$. Then $c = (c - (a \wedge c)) + a \wedge c = a \wedge b + w + v + (b - (a \wedge b))$ and $c + (a - (a \wedge c)) = a \wedge c + w + v + (b - (a \wedge b)) + (a - (a \wedge c)) \in E$. Hence $c - b = w + v$ and $a = w + [(a \wedge b) + (a - (a \wedge c))]$ which concludes $a \leftrightarrow (c - b)$ while $w + v + (a \wedge b) + (a - (a \wedge c)) \in E$.

Applying (DMP), $(c - b) \wedge a \in E$ so that by Proposition 2.5, we have $a \overset{c}{\longleftrightarrow} (c - b)$. □

A maximal set of mutually strongly compatible elements of E is said to be a *block* of E. For example, if E is an MV-algebra, then it is a unique block of E.

Let $\{E_t\}_{t\in T}$ be a system of effect algebras such that $E_t \cap E_s = \{0, 1\}$ for $t \neq s$. The set $E := \bigcup_{t\in T} E_t$ can be organized into an effect algebra such that $x + y$ is defined in E iff $x, y \in E_t$ for some $t \in T$ and if $x + y$ is defined in E_t, in such a case, $x + y$ taken in E is equal to that taken in E_t. Then E is an effect algebra which is said to be a *horizontal sum* of the system of effect algebras $\{E_t\}_{t\in T}$.

If E is a horizontal sum of a system of MV-algebras $\{E_t\}_{t\in T}$, then E is not necessary an MV-algebra, and $\{E_t\}_{t\in T}$ is a system of all blocks in E.

Theorem 3.2. *Every block of an effect algebra E with* (*RIP*) *and* (*DMP*) *is an effect subalgebra of E which is an MV-algebra. Moreover, any such effect algebra is a set-theoretical union of its blocks.*

Proof: Let M be a block of E. Then $0, 1 \in M$. Assume $a, b, c \in M$. By Proposition 3.1, $a \overset{c}{\longleftrightarrow} b'$ so that $b' \in M$. If $b + c \in E$, we have by Proposition 3.1, $a \overset{c}{\longleftrightarrow} (b' - c)$, consequently $a \overset{c}{\longleftrightarrow} (b' - c)' = b + c$ which proves that M is an effect subalgebra of E. Moreover, if $x, y \in M$, by Propositions 2.9 and 2.10, $x \vee y, x \wedge y \in M$.

In view of (iii) of Theorem 2.8, we have $(x \vee y) - y = x - (x \wedge y)$ for all $x, y \in M$ which is a necessary and sufficient condition in order M to be an MV-algebra.

If A is any subset of mutually strongly compatible elements of E, due to Zorn's lemma there exists a block of E containing A. In particular, by Proposition 3.1, the set $A = \{0, a, a', 1\} (a \in E)$ is a set of mutually strongly compatible elements of E. This proves that E is a set-theoretical union of its blocks. □

Theorem 3.3. *Every effect algebra E satisfying* (*RIP*) *and* (*DMP*) *is a set-theoretical union of MV-algebras.*

Proof: It follows from Theorem 3.2. □

If E satisfies only (RIP), then we have the following result:

Theorem 3.4. *Every effect algebra E satisfying* (*RIP*) *is a set-theoretical union of blocks which are distributive sublattices of E.*

Proof: Because of Propositions 2.9 and 2.10, each block of E is a distributive sublattice of E, and E is a set-theoretical union of its blocks. □

We recall that Theorem 3.2 and Theorem 3.3 generalize the result of Riečanová (2000a) who proved that for lattice-ordered effect algebras. On the other hand, Example 2.3 gives an effect algebra E where E is a unique block which however is not an MV-algebra. Example 2.1 shows that its blocks are not MV-algebras even if E satisfies (RDP).

Problem. Now we formulate the following problem: Characterize those effect algebras which can be covered by MV-subalgebras. We recall that any lattice-ordered effect algebra, any effect algebra satisfying (RIP) and (DMP) can be covered by MV-subalgebras. In addition, any orthomodular poset and any orthoalgebra has such a covering property (for the later two cases, blocks are precisely Boolean-algebras).

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